

3.9b der of logarithms

def $y = \log_a x$ means $a^y = x$

properties

$$\log(a \cdot b) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^x = x \log a$$

$\ln x = \log_e x$ **natural** $\ln e = 1$

$\log x = \log_{10} x$ **common** $\ln e^x = x$

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$$\frac{d}{dx} \ln x = \frac{1}{x}$$

der of outside

der of inside

$$\frac{d}{dx} \ln(\sin x) = \frac{1}{\sin x} \cdot \cos x$$

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \cdot \frac{du}{dx} = \frac{u'}{u}$$

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$$\frac{d}{dx} \frac{\ln(x^2 + 2x)}{e^{3x}} = \frac{e^{3x} \cdot (2x+2) \cdot \frac{1}{x^2+2x} - \ln(x^2+2x) \cdot e^{3x}}{(e^{3x})^2}$$

$$\frac{d}{dx} \log x = ? \quad \frac{1}{x \ln 10}$$

change of base $\log_a x = \frac{\log_b x}{\log_b a}$

or $\log x = \frac{\ln x}{\ln 10}$

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$$\frac{d}{dx} \log_a u = \frac{du}{dx} \cdot \frac{1}{u \ln a}$$

$$\frac{d}{dx} \log(\tan x) = \sec^2 x \cdot \frac{1}{\tan x \cdot \ln 10}$$

$$\frac{d}{dx} \log_5 x^2 = 2x \cdot \frac{1}{x^2 \ln 5} = \frac{2}{x \ln 5}$$

\downarrow

$$\frac{d}{dx} 2 \log_5 x = 2 \cdot \frac{1}{x \ln 5}$$

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logarithmic differentiation

Ex 1. $y = x^x$ find $\frac{dy}{dx}$

① take \ln of both sides

② use properties of \ln to simplify

③ deriv.

④ solve for $\frac{dy}{dx}$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{dy}{dx} \cdot \frac{1}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = (1 + \ln x) y$$

$$= (1 + \ln x) \cdot x^x$$

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